of ξ. The distribution of magnetic field intensity H_x is given in Fig. 4 [1) $\xi = 0.0125$; 2) $\xi = 4.137$, $U_e = 0.2 \cdot 10^5$ cm/sec]; and μ, λ, ν_H are considered to be power series functions of the density and temperature.

The above program was chosen because of the capabilities of the M-222 computer memory. It may seem that the use of matrix forcing for simultaneous computation of all the desired quantities would lead to more rapid solution of the problem. It should be noted that, while the accuracy of the increase of the vertical velocity component is not important in calculating the unsteady boundary layer for an incompressible liquid, and the results vary only by a few percent when this is totally omitted; nevertheless, this component requires accurate computation for a compressible liquid.

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INTERNAL RESONANCES IN HYDRODYNAMICS

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§1. In the theory of the vibrations of systems close to linear [1], internal resonance is defined as the proportionality of several natural frequencies to natural numbers. The present article discusses internal resonances in hydrodynamics.

In the case of internal resonance, forced vibrations of small amplitude, brought about by a harmonic perturbation, can differ considerably from harmonic. An example is discontinuous vibrations of a gas (shock waves), observed in a closed tube with a harmonic motion of a piston [2, 3].

Autovibrations of small amplitude can also be essentially nonharmonic, for example, autovibrations in a low-pressure gas discharge [4].

The main features of resonances come out with the consideration of the boundary-value problem for the real vector X:

$$\frac{\partial X}{\partial t} + L_1 X + L_2 X^2 + \ldots = \sum_{k=1}^{n} \varepsilon_k C_k e^{i\omega_k t} + \text{ c.c.}, \quad UX = 0 \tag{1.1}$$

(c.c. is an expression, complex-conjugate to the preceding). Here the real coefficients L and the matrix U in the boundary condition can depend on the coordinates x and are polynomials with respect to $D = \partial/\partial x$. The region of change of x is assumed to be bounded. Each perturbation with the frequency $\omega_k > 0$ and the form $C_k(x)$ is proportional to a small amplitude ε_k. The frequencies ω_k and their differences are assumed to be fairly great (the effects of the type of slow change in the parameters are not taken into consideration here). It is postulated that the problem

$$pX + L_1 X = 0, \; UX = 0 \tag{1.2}$$

has several simple eigennumbers $p = \gamma + i\Omega$, with small increments of γ and positive frequencies Ω. Let these be the numbers p_m ($m = 1, 2, \ldots, M$), the corresponding eigenfunctions

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X_m, and the eigennumbers of the conjugate problem Z_m; the increments of the other eigenvalues are negative and fairly large.

Since the numbers p_m are almost imaginary, they are approximately proportional to some natural numbers. The conditions with which such a proportionality is real are discussed below.

It is obviously always possible to select whole numbers a_{mk}, b_m so that

$$\Omega_m \approx (a_{m1}\omega_1 + \ldots + a_{mn}\omega_n)/b_m \equiv \nu_m.$$

The resonance vibrations characterized by these numbers correspond to the solution

$$X = \left(\sum_{m=1}^{M} X_m Q_m \exp i\nu_m t + \sum_{k=1}^{n} Y_k \varepsilon_k \exp i\omega_k t \right) + \text{c.c.}, + \ldots \tag{1.3}$$

in the form of a power series in terms of values of $Q_m \exp i\nu_m t$, $\varepsilon_k \exp i\omega_k t$ and the complex-conjugate values, with coefficients depending only on x. The equation for the amplitude of $Q_m(t)$ is also sought in the form of a series in terms of values of Q_r, ε_k and the conjugate values. This equation has the form

$$dQ_m/dt = Q_m(\delta_m + \ldots) + \sum \varepsilon_\alpha^a \bar{\varepsilon}_\beta^b Q_s^c \bar{Q}_r^d \ldots (p_{m\alpha\ldots} + \ldots) \tag{1.4}$$
$$(\delta_m = p_m - i\nu_m,\ m = 1, \ldots, M),$$

where series in powers of $|Q_r|^2$, $|\xi_k|^2$ stand in parentheses; the sum is taken over all the natural numbers a, b, c, . . . , satisfying the nonidentical equalities

$$\nu_m = a\omega_\alpha + \ldots - b\omega_\beta - \ldots + c\nu_s + \ldots - d\nu_r - \ldots .$$

The identity $\nu_m = \nu_m$ corresponds to the series standing before the sum; the terms of this series describe nonresonance effects, existing with any given values of ν_m.

As in the case of autonomous systems [5] with M = 1, the coefficients in (1.3) are determined consecutively from linear inhomogeneous boundary-value problems, obtained after substituting (1.3), (1.4) into (1.1). The coefficients in (1.4) are determined from the condition of the boundedness, as $\delta_m \to 0$, of the corresponding coefficients in (1.3). This condition has the form

$$\langle \Psi \cdot Z_m \rangle \equiv \int (\Psi \cdot \bar{Z}_m)\, dx = 0, \tag{1.5}$$

where Ψ is a free term of the inhomogeneous problem, depending linearly on the sought coefficient in (1.4); the integration is carried out over the region of change in x.

To find the coefficient $p_{m\alpha}$. . . of Eq. (1.4), in (1.1) it is necessary to take account of all the terms of the series with powers $\leqslant N = a + b + c + \ldots$; to find the greatest nonresonance terms in (1.4), it is sufficient to retain the quadratic and cubic terms in (1.1). From this it can be seen that the resonance effects are considerable if the order of the resonance $N \leqslant 3$. Nonresonance effects predominate* if the order of the resonances N > 3.

In simple examples, it is possible to evaluate the steady-state solutions of (1.4) and, by the same token, to clarify the effect of resonances on the amplitude (but not on the stability) of the vibrations. Let, for example, in (1.1), (1.2) the numbers n = 1, M = 2; here $p_2 = i\Omega_2 = 9p_1$, $\Omega_1 = \omega$; then, in vibrations with the frequency ω, resonance of the ninth order is insignificant, since $Q_2 \sim Q_1^7 \ll Q_1 \sim \varepsilon^{1/3}$. If, in distinction from the preceding case, there is a third number $p_3 = 3p_1 = p_2/3$, then all three amplitudes $\sim \varepsilon^{1/3}$.

Above, for determinacy, problem (1.1) has been discussed with a linear homogeneous boundary condition. It is frequently possible to bring problems with other conditions into the form (1.1) by the introduction of new variables (for example, in the case of a linear inhomogeneous condition, $X = X^\circ + A$ is used, where A is some function, satisfying the inhomogeneous condition). However, such an approximation is not needed. For each type of resonance

*Exceptional cases are possible [for example, where some coefficients are equal to zero in (1.4)].

the series (1.3), (1.4) do not change their structure, if the equations and boundary conditions for X are represented in the form of power series in terms of the components of X, their derivatives with respect to t, x, and the perturbations $\varepsilon_k \exp(i\omega_k t)$. For the coefficients in (1.3) there will be obtained problems with inhomogeneous conditions; homogeneity of the conditions can be achieved (if necessary) by the method noted above.

Problems with nonanalytical nonlinearities (for example, of the kind $X_n|X_m|$) require a special discussion.

To bring out the principal laws of resonance, it is sufficient to retain only the largest terms in (1.4). In this basic approximation, the frequency differences δ_m (by definition, the resonances are small) are taken into account only by the terms $Q_m\delta_m$; in the coefficients of the remaining terms, it is assumed that $p_m = i\Omega_m = i\nu_m$ (m = 1, ..., M), so as not to go beyond the accuracy of the approximation. In the examples given above, the resonances are considered in the basic approximation.

§2. As an example, let us consider the problem of finding X = (ξ, w) from the equations

$$\xi^{\cdot} + w' = 0,\ w^{\cdot} + \xi' + \Phi' = 0\ (0 \leqslant x \leqslant 1),$$
$$\Phi = w^2/\rho + (\rho^\beta - 1)/\beta - \xi = w^2 + 1/2(\beta - 1)\xi^2 + \ldots \tag{2.1}$$

and the boundary conditions

$$(w)_0 = \xi l^{\cdot},\ (w)_1 = 0. \tag{2.2}$$

Here a dot indicates differentiation with respect to t and a prime, with respect to x. Written in dimensionless form (so that, with ε = 0, the length of the tube, the density $\rho = 1 + \xi$, the pressure ρ^β, and the speed of sound are equal to unity), the problem (2.1), (2.2) describes the vibrations of a gas in a closed tube, excited by the motion of a piston.* The form of the displacement of the piston

$$l(\omega t) = \sum l_m \exp(im\omega t) \quad (l_{-m} = \bar{l}_m,\ l_0 = 0) \tag{2.3}$$

can be (in distinction from [6, 7]) anharmonic. The natural frequencies of the linear homogeneous problem (2.1), (2.2) are equal to πm, where m is a whole number; therefore, there exists an infinite number of internal resonances of the second order.

The expansions (1.3), (1.4) for the problem (2.1), (2.2) are found in the form

$$X = A_1 + A_2 + A_3 + \ldots,\quad A_1 = \sum Q_m X_m e^{im\omega t},$$
$$X_m = (\cos m\pi x, -i\sin m\pi x),\ Q_{-m} = \bar{Q}_m, \tag{2.4}$$
$$dQ_m/dt = im(\pi - \omega)Q_m + a_m + \ldots .$$

Here the sum is taken over all whole numbers m; the eigenfunctions X_m correspond to the natural frequency πm. It is convenient to determine the series (2.4), assuming that $Q_m \sim \varepsilon^{1/2}$, $\omega - \pi \sim \varepsilon^{1/2}$, $A_m \sim \varepsilon^{m/2}$; the coefficients $a_m \sim \varepsilon$ are determined from the condition of the boundedness of A_2.

The vector A_2 is determined from the conditions (2.2) and the equation

$$A_2^{\cdot} + BA_2' + \Phi_2' E + \sum a_m X_m \exp im\omega t = 0, \tag{2.5}$$
$$B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ E = \begin{pmatrix} 0 \\ 1 \end{pmatrix},\quad \Phi_2 = w_1^2 + (1/2)(\beta - 1)\xi_1^2,$$

where ξ_1, w_1 are the components of A_1.

*Replacement of the exact solution for the piston ($w = \rho\varepsilon l^{\cdot}$ with $= \varepsilon l$) by the approximation (2.2) does not decrease the exactness of the basic approximation (see Sec. 1).

Setting in (2.2), (2.5)

$$A_2 = \sum (Y_m + \varepsilon l_m T_m) e^{im\omega t},$$
$$\Phi_2 = \sum F_m e^{im\omega t}, \quad T_m = [1, \; im\omega(1-x)], \tag{2.6}$$

we obtain for Y_m the homogeneous conditions (2.2) and the equation†

$$im\omega Y_m + BY_m' + \Psi = 0, \quad \Psi = a_m X_m + E\left[F_m' - m^2\omega^2 l_m \varepsilon (1-x)\right]. \tag{2.7}$$

As $\omega \to \pi$, the value of Y_m is finite if condition (1.5) is satisfied. In the basic approximation used (see Sec. 1), it is sufficient to find a_m with $\omega = \pi$. From (1.5), (2.5)-(2.7), taking account of the equality $Z_m = X_m$, there is obtained

$$a_m = -i\int_0^1 \left[F_m' - m^2\pi^2 l_m \varepsilon (1-x)\right] \sin m\pi x dx = i\pi m \quad \left[\varepsilon l_m + (1/8)(1+\beta) \sum_n Q_n Q_{m-n}\right]. \tag{2.8}$$

With m = 0, from (2.4), (2.8) it follows that $Q_0 = 0$ (in accordance with the conservation of the total mass of gas in the tube). For other stationary amplitudes, after introduction of the notation

$$\mu^2 = 8\varepsilon/(1+\beta), \; f_0 = 4(1-\omega/\pi)/\mu(1+\beta); \; f_m = Q_m/\mu \;\; (m \neq 0)$$

the following equations are obtained:

$$l_m + \sum_n f_n f_{m-n} = L\delta_{0m} \quad (m = 0, \pm 1, \ldots), \tag{2.9}$$

in which L is a positive parameter. The dependences of the amplitudes f_m on the relative frequency difference f_0 is most conveniently found in parametric form, determining the amplitude and the frequency difference from (2.9) as functions of L.

Equations (2.9) are equivalent to the equation

$$f^2 = L - l(\theta) \tag{2.10}$$

for the real nonperiodic function

$$f(\theta) = \sum f_m e^{im\theta}. \tag{2.11}$$

The form of the displacement of the piston $l(\theta)$ is determined in (2.3). The frequency difference f_0, the parameter L, and the solution X are expressed in terms of f in the form

$$f_0 = \frac{1}{2\pi}\int_0^{2\pi} f d\theta, \quad L = \frac{1}{2\pi}\int_0^{2\pi} f^2 d\theta; \tag{2.12}$$

$$X \approx A_1 = (1/2)\mu(f_+ + f_- - 2f_0, \; f_- - f_+), \;\; f_\pm = f(\omega t \pm \pi x). \tag{2.13}$$

The results of [6, 7] are expanded for the case of an anharmonic discontinuous form of l in the following manner. Let $l(\theta) \leqslant 1$; then the continuous solution of (2.10)

$$f = (f_0/|f_0|)|L - l|^{1/2} \;\; (L \geqslant 1, \; |f_0| \geqslant f_* = f_0(1)) \tag{2.14}$$

is singular and corresponds to the frequency difference $f_0(L)$, determined from (2.12).

With $|f_0| < f_*$, only discontinuous solutions of (2.10) are possible. The only solutions of interest are those describing compression shock waves [2] (after the passage of a compression wave, the density at a given point rises). In such admissible solutions, in accordance

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†Since the value of Y_m does not depend explicitly on t and is quadratic with respect to $Q_n(t)$, $Y_m \sim A_2$ and therefore does not enter into (2.7).

with (2.13), the function $f(\theta)$, with a rise in θ, can increase jumpwise, but cannot decrease jumpwise.†

If, in the interval $0 \leqslant \theta < 2\pi$, the value $l = 1$ is assumed at the singular point $\theta = \theta_0$, then the admissible solution

$$f = |1 - l|^{1/2} \begin{cases} -1 & \theta_0 \leqslant \theta < \varphi \\ 1 & \varphi < \theta \leqslant \theta_0 + 2\pi \end{cases} \tag{2.15}$$
$$(0 \leqslant \varphi \leqslant 2\pi, \ |f_0| \leqslant f_*)$$

is singular and corresponds to the frequency difference $f_0(\varphi)$, determined from (2.12).

An arbitrary frequency difference corresponds to one of the solutions (2.14), (2.15). Figure 1 shows the form of these solutions near $f_0 \approx f_*$ for

$$l(\theta) = \alpha \cos\theta + (1 - \alpha)\cos 3\theta \ (0 < \alpha \ll 1, \ l \leqslant l(0) = 1). \tag{2.16}$$

The curve of $f(\theta, f_*)$ is shown by the points.

In addition to (2.14), (2.15), with a given frequency difference, there exists an infinite set of discontinuous solutions of (2.10); they all contain at least one rarefaction shock wave and, consequently, are not admissible.

If the value $l = 1$ is attained in the interval $[0, 2\pi]$ at several points, then, with $|f_0| < f_*$, there exists an infinite set of admissible solutions, differing in the number, value, and mutual arrangement of the shock waves. Figure 2 shows several solutions 1-4 for $\alpha = 0$ in (2.16) and $f_0 = 0$.

In the experiments of [3, 6, 7], only symmetrical vibrations of type 1 were observed. A possible reason is the instability of other types of vibrations. Another possibility is the dependence of the type of vibrations on the initial conditions and (or) on the means used for varying the parameters. Let, for example, at the initial moment there be given the form (2.16) and the frequency difference $f_0 > f_*$, so that there exist continuous vibrations (see Fig. 1). Then vibrations of type 2 are obtained if first f_0 and then α are slowly decreased to zero. If $f_0 \to 0$ with $\alpha = 0$, then, as observations [3, 6, 7] have shown, out of all those possible, there arise vibrations of type 1.

It must be noted that shock waves heat the gas; this leads to an observed [3] increase in the resonance frequency (with which $f_0 = 0$). Since the evolution of heat in each shock wave is proportional to the cube of the value of the discontinuity [2], the heating and the increase in the frequency are proportional to μ^3. The coefficient of proportionality depends on the heat-transfer conditions and on the type of vibrations; with $f_0 \approx f_*$, vibrations of type 1 lead to the least heating.

§3. Let us consider the problem (2.1), (2.2), in which the second boundary condition is replaced by the approximate condition for an open tube $(\xi)_1 = 0$. For simplicity, we assume that $\beta = 1$.

In the given case, there is an infinite number of internal resonances of the third order. The solution is represented by the expressions (2.4), in which π is replaced by $(1/2)\pi$, and m are odd numbers. Vibrations will be considered in which $Q_m \sim \varepsilon^{1/3}$, $A_m \sim \varepsilon^{m/3}$, $\omega - (1/2)\pi \sim \varepsilon^{2/3}$. The coefficients $a_m \sim \varepsilon$ are determined from the problem for A_3. Preliminarily, it is required to find $A_2 = (\xi_2, w_2)$ from homogeneous conditions and Eq. (2.5), in which the sum is dropped; with $\omega = (1/2)\pi$,

$$w_2 = -\frac{i}{8}\sum_{n,s} Q_s Q_n e^{ia\omega t}\left[\left(\frac{b^2}{ns} - 2\right)\sin a\eta + \frac{ab}{ns}\sin b\eta + 2a\eta\cos a\eta\right] \tag{3.1}$$
$$(\eta = 1/2 \cdot \pi x, \ a = n + s, \ b = n - s).$$

Setting

†The remaining relationships at a discontinuity [2], in the approximation (2.13), reduce to acoustical and are satisfied automatically.

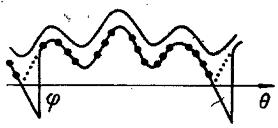

Fig. 1

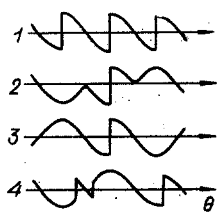

Fig. 2

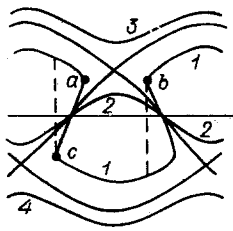

Fig. 3

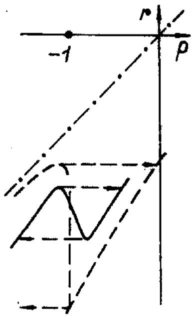

Fig. 4

$$A_3 = \sum (Y_m + im\omega\varepsilon l_m E)\, e^{im\omega t}, \quad 2w_1 w_2 - w_1^2 \xi_1 = \sum F_m e^{im\omega t}, \tag{3.2}$$

for Y_m, with an odd value of m, we obtain homogeneous conditions and Eq. (2.7), in which the factor $(1 - x)$ is dropped. From (1.5), (2.7), (2.1), (3.1), (3.2), with $\omega = \pi/2$, $Z_m = X_m$, we obtain

$$a_m = \frac{1}{2} i\pi m\left[\varepsilon l_m - \frac{5}{16}\left(Q_m \sum |Q_n|^2 + \frac{1}{3}\sum_{s,n} Q_s Q_n Q_{m-n-s}\right)\right]. \tag{3.3}$$

After introduction of the notation

$$\mu^3 = -(24/5)\varepsilon, \quad f_m = Q_m/\mu, \quad r = (16/5)(2\omega/\pi - 1)/\mu^2 \tag{3.4}$$

for the stationary amplitudes, from (2.4), (3.3) it follows that

$$\sum f_s f_n f_{m-n-s} + 3f_m\left(\sum |f_n|^2 + r\right) + 2l_m = 0. \tag{3.5}$$

For the function f in (2.11), from (3.5) it follows that

$$f^3 + 3pf + 2q(\theta) = 0, \tag{3.6}$$

where q is obtained from (2.3), discarding even harmonics, and

$$p = \frac{1}{2\pi}\int_0^{2\pi} f^2 d\theta + r. \tag{3.7}$$

Since f contains only odd harmonics, the sought solutions of (3.6) must satisfy the condition $f(\theta + \pi) = -f(\theta)$. For X, (2.13) holds, where $f_0 = 0$.

In what follows, it is assumed that $q = l = \cos\theta$. Figure 3 shows the roots of Eq. (3.6) for $0 > p > -1$ (curve 1), $p < -1$ (curves 2-4), and $p = -1$. With $P \geqslant 0$, curve 1 is singular and represents the solution f. With $p < -1$, the solution f is represented by curve 2. With $-1 < p < 0$, all the singular solutions $f(\theta)$ are discontinuous and contain rarefaction waves (see Sec. 2); the discontinuities in one of the possible solutions are shown by the dashed line in Fig. 3.

The absence of solutions containing only compression waves indicates that in the present problem, with a decrease in the viscosity and the thermal conductivity, the width of the front in the shock wave remains finite and is determined by some physical effect which does not bring about heating of the gas. Such an effect is radiation from the open end of the tube.

To take account of radiation, in Eq. (2.4) for Q_m, $(1/2)\pi m$ must be replaced by the natural frequency, changed by the radiation. The change comes down [8] to multiplication of $(1/2)\pi m$ by

$$1 - c_1 R + imc_2 R^2. \tag{3.8}$$

Here $c_{1,2} \sim 1$ are positive constants (for example, $c_1 \approx 8/(3\pi)$, $c_2 \approx \pi/4$ for a tube with a flange [8]); it is postulated that the radius of the tube $R \ll 1$.

Taking account of (3.8) leads to the replacement of (3.4), (3.6) by

$$r = (16/5\mu^2)(2\omega/\pi - 1 + c_1R),\ f^3 + 3pf + 2q = \chi df/d\theta \quad (\chi = 48R^2c_2/5\mu^2). \tag{3.9}$$

The effects of radiation are small in comparison with nonlinear effects with $\kappa \ll 1$, which is assumed to be satisfied.

An examination of the field of the directions of Eq. (3.9) shows that, with $s = p + 1 \gg \chi$, the periodic curve of f is close to the discontinuous curve in Fig. 3 (the width of the "discontinuity" is $\sim \kappa$). The integral curve, departing from c and intersecting the straight line ab at the point θ, has the slope $f' = 2(\cos\theta - \cos\theta_a)/\chi \sim 1$; from this it follows that $\theta - \theta_a \sim \chi \ll \theta_a$. Intersection with $\theta = 0$ and tangency at the point b takes place with values $s \sim \chi$, where the discontinuous periodic curve is deformed into curve 2.

The frequency dependence r(p) is determined from (3.7). Figure 4 shows qualitatively the dependence r(p) with $\chi > 0$, with $\chi = +0$, and the asymptote $r = p$ (solid, dashed, and dashed-dot lines, respectively). The distance between the extrema of the curves $\sim \chi$. The values of p, where $dr/dp < 0$, are not realized; the directions of the shock waves p(r) are shown by arrows. In addition to the amplitude, with discontinuities of the parameter p, the form of the vibrations changes considerably.

The above discussion can be extended without ambiguity to the case of an anharmonic form of $q(\theta)$. Specifically, if q changes sign at the point θ_0, then the number of discontinuities of $f(\theta, p, \chi = +0)$ in the interval $[\theta_0, \pi + \theta_0]$ is equal to the number contained in the $[\theta_0, \pi + \theta_0]$ intervals in which $q^2 + p^3$ changes sign and q does not change [for example, for (2.16), this number is equal to zero, unity, or three].

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